TIME-TO-START (A)

"George, I have an order from Seldon Fans for a 100 h.p., 3600 rpm totally enclosed motor to drive a fan. They're worried about starting time. Could you work out the starting time. We have to call them back this afternoon and confirm the order." Kirkland, the *sales manager, put this question to George Boone, one of the design engineers at Maplectric.* *

*Names in this case have been disguised.

TIME-TO-START (A)

Maplectric was a small electric motor manufacturer with
about 800 employees. It manufactured a full line of NEMA
motors and a few sizes beyond the standard. It also manu-
factured a line of D.C. motors. In production and sales
it had to compete against the larger manufacturers. Be-
cause of its vulnerability it could not expect to compete
with them by cutting prices. Therefore Maplectric devel-
oped a policy of going after the business in a selected
area and with specific customers by providing excellent
service and rapid delivery. Being a small firm they could
respond more quickly to customer's demands than the larger
competitors.

The president and owner did not believe in advertising and
depended solely on contact through salesmen. He felt that
for the cost of even a modest advertising campaign, he
could hire an extra salesman who would bring in more busi-
ness. This policy could not be disputed for the company
had grown steadily from its founding, not spectacularly
but steadily.

George had joined the firm two years earlier with a Bache-
lor's degree in mechanical engineering and two years of
unrelated experience. He and 11 others in the engineering
department reported to the Chief Engineer. Two of the
others had engineering degrees roughly the same vintage as
George's but in electrical engineering. The remainder of
the staff were draftsmen.

Everyone worked at the drafting board on the mechanical
design of the various motors. The work was distributed
equally among the members of the department based upon
their availability and proven skills. The electrical de-
sign of the motors was done by two consultants. As Bob
Linder, the general manager, said when hiring George, "The
electrical design can be written down on one sheet of 8-1/2
x 11 paper, the rest is mechanical." This was literally
true. The electrical specifications were contained on a
standard 8-1/2 x 11 sheet crammed with information.

At Maplectric George had established a reputation for being
willing and capable of tackling unusual problems in mechanics.
Therefore, Kirkland naturally brought this problem to him.

"Why do they need the information?" asked George.

"They'd like to be able to start the motor directly on
line. If the starting time is too great they will have to
use reduced voltage starting to prevent burn out. This,

of course, would mean a considerable difference in installation cost," replied Kirkland.

Then George and Kirkland got together and established the exact specification. A standard 100 h.p., 405 TS frame machine was to be used to drive Seldon's fan which required 85 h.p. at 3520 rpm. The motor was a standard squirrel cage type: 3 phase, 60 cycle, 440 volts, NEMA Design B. The WR^2 of the fan was 90 lbs. ft.^{2}. The moment of inertia in the electrical industry is referred to as WR^2 where W is the weight of rotating parts and R is the radius of gyration.

NEMA specifications set industrial standards for electric motor configuration and performance (Exhibit A-1). Typical speed torque curves are given in the specification.

After Kirkland had left, George hauled out the electrical specification and drawings for the Maplectric motor. From the electric specification he found that their design actually had 110% starting torque and 200% breakdown torque. The full load speed was 3520 rpm. This was typical of the machines designed by one of their consultants. The starting torque was higher than specification with starting current right on specification. The other consultant usually designed with starting torque right on specification but starting current less than permitted.

The squirrel cage induction motor (Exhibit A-2) is of simple design. The only rotating parts are the shaft and the rotor. From the drawings George was able to quickly compute that the rotor weight was 317 lbs. with an R^2 of 18 in.^{2} and shaft weight was 59.4 with an R^2 of 1.1 in.^{2}. Before starting George remembered that he had done a similar calculation a year earlier. Thumbing through his files he found his calculations. It was for a 150 h.p. 900 rpm motor driving a machine with attached WR^2 of 1950 lb/ft.^{2} but no power required during starting. This machine had a rotor WR^2 of 285 lb/ft.^{2}

In the previous case it had been a fairly simple calculation and he had simplified the problem by calculating the upper and lower bounds to the starting time.

The Seldon calculations would be more difficult than the above but it would be worth reviewing this before startin on the Seldon problem.

SELECTION of an electric motor includes a choice of motor type and motor design, along with a choice of the proper size. The objective of informed motor selection is to arrive at the safest possible installation consistent with minimum cost, horsepower, and frame size for the specified life expectancy, load torque, load inertia, and duty cycle.

Each electrical and mechanical feature of a motor is intended for a particular type of application. Variations in possible combinations of enclosures, mountings, bearings, installation, and electrical characteristics, often seem endless. Therefore, selection of the optimum motor seems a quite complicated task. This is not necessarily so; when all factors are considered one *best* motor usually emerges.

Motor Standards

Standards for motors fall into three general categories: environmental, electrical, and mechanical.

Environmental

Motor manufacturers and standardization agencies have agreed upon a standard, or "normal," environment. Most motor applications fall within the envelope defined by these standards, and all standard off-the-shelf motors are designed to operate satisfactorily under these conditions. The standard environment is:

■ Ambient temperature between 10 and 40 C.

■ Altitude not above 3,300 ft, nor below sea level, nor in a pressurized or evacuated space which results in pressure outside these limits.

■ Installation on a rigid surface.

■ Location which allows free and unrestricted circulation of clean, dry, cooling air.

■ Location providing access for periodic inspection, lubrication, and maintenance.

Temperature Rise: The relationship of class of insulation, use of service factor, temperature rise, and total temperature is shown in Table 2, Chapter 2. All temperature rises given are maximum, and motor manufacturers often supply motors well under these limiting figures. The user, however, must be prepared to cope with the maximum condition.

Electrical

Locked-Rotor Current: This is the current which the motor draws at rated voltage and frequency when the rotor is not turning. Since the installation wiring, fusing, controls, and power source are exposed to this current at the instant of starting, the value of the locked-rotor current must be known.

NEMA standards, ANSI standards, and the National Electrical Code (NEC) require that the ratio of starting kva to rated hp be designated on the motor nameplate. This requirement applies to all a-c induction motors of 1 to 500 hp regardless of design, phase, voltage, or frequency rating, except polyphase wound-rotor motors. These ratings are usually given on the nameplate in code form, Table 1.

Mechanical

Motor performance is best expressed by the relationship of speed and torque. To apply a motor to a machine or piece of equipment, the speeds and torques required by that machine must be known. When motor speed is plotted versus motor torque, performance of the drive can be visualized. It is also possible to predict how changes in load or torque will affect performance.

Torque: Motors are rated in terms of horsepower and speed. From these data, full-load torque can be determined by $T = 5,252\ P/N$, where T = full-load torque, lb-ft; P = power, hp; and N = motor speed, rpm. While full-load torque is a basic indication of the size of the motor, other characteristics (such as ability to start a load from rest, ability to accelerate a load to full running speed, and maximum overload to cause an abrupt loss of speed) are important and have been carefully defined.

Horsepower and Speed: The synchronous speed of an a-c motor can be calculated from $N_s = 120\ f/p$, where N_s = synchronous speed, rpm; f = frequency, Hz; and p = number of poles. Thus, a 60-Hz, 4-pole motor has a synchronous speed of 1,800 rpm. Table 4 shows the standardized horsepower assignments and synchronous speeds of integral-horsepower a-c motors.

Short-Time Rating: Many motor loads are of short duration, or have a different magnitude in each cycle. It may not be necessary or economical to use continuous-duty motors for these loads. Typical short-time loads include cranes, hoists, saws, grinders, valve operators, and door or window operators. Standard short-time duty (intermittent-duty) ratings for motors for these applications are 5, 15, 30, or 60 min. For time ratings longer than 60 min, continuous-duty motors should be used.

Enclosures: The enclosure protects the vital working parts of the motor, such as bearings and windings, from contaminants in the ventilating air. In single-phase motors, the governor and switching mechanism for the phase circuit are also protected. The enclosure also protects the operator or the driven machinery from damage if the motor fails. Descriptions of enclosures, motor dimensions, and frame numbers are given in Chapter 1. Many variations in enclosures, flange mountings, and shaft configurations have been standardized to be consistent with the basic open-motor dimensions.

Standard Frame Assignments: Frame configurations for motors have been standardized by NEMA, and a standard frame numbering system is used. Each code number—known as a *frame assignment*—completely defines the motor size and mounting dimensions. Available frame asignments for single-speed polyphase motors are shown in Table 5; Table 6 lists the assignments for single-speed, single-phase motors.

Table 1—Locked-Rotor kva/hp Code Designations*

Code Letter	Locked-Rotor kva/hp	Code Letter	Locked-Rotor kva/hp
A	0-3.15	L	9.0-10.0
B	3.15-3.55	M	10.0-11.2
C	3.55-4.0	N	11.2-12.5
D	4.0 -4.5	P	12.5-14.0
E	4.5 -5.0	R	14.0-16.0
F	5.0 -5.6	S	16.0-18.0
G	5.6 -6.3	T	18.0-20.0
H	6.3 -7.1	U	20.0-22.4
J	7.1 -8.0	V	22.4 and up
K	8.0 -9.0		

From NEMA MG 1-10.37 and NEC 430-7(b).

*Except polyphase wound-rotor motors.

For motors with dual ratings, the following rules determine the code letter to be used:

Motor Type	Code Letter Corresponding to kva/hp for
Multispeed	
Variable torque	Highest speed
Constant torque	Highest speed
Constant horsepower	Highest kva/hp
Wye delta, starting on wye	Wye connection
Dual voltage	Highest kva/hp
Dual frequency, 60/50 Hz	60-Hz kva/hp
Part-winding start	Full-winding kva/hp

EXHIBIT A-1

Table 4—Standard 60-Hz Horsepowers and Speeds

Rating (hp)	Synchronous Speed (rpm)
½	514, 600, 720, 900
¾	514, 600, 720, 900, 1200
1	514, 600, 720, 900, 1200, 1800
1½, 2, 3, 5, 7½, 10, 15, 20, 25, 30, 40, 50, 60, 75, 100, 125	514, 600, 720, 900, 1200, 1800, 3600*
150	600, 720, 900, 1200, 1800, 3600*
200	720, 900, 1200, 1800, 3600*
250	900, 1200, 1800, 3600*
300, 350	1200, 1800, 3600*
400, 450, 500	1800, 3600*

From NEMA MG 1-10.32.
Synchronous speeds shown in lightface are for polyphase motors only; those in boldface are for both single and polyphase motors.
*Does not apply to polyphase wound-rotor motors.

Table 5—Frame Assignments for Continuous-Duty, 60-Hz, Polyphase Motors

Rating (hp)	3600 Open	3600 TEFC	1800 Open	1800 TEFC	1200 Open and TEFC	900
½	NA	NA	NA	NA	NA	143T
¾	NA	NA	NA	NA	143T	145T
1	NA	NA	143T	143T	145T	182T
1½	143T	143T	145T	145T	182T	184T
2	145T	145T	145T	145T	184T	213T
3	145T	182T	182T	182T	213T	215T
5	182T	184T	184T	184T	215T	254T
7½	184T	213T	213T	213T	254T	256T
10	213T	215T	215T	215T	256T	284T
15	215T	254T	254T	254T	284T	286T
20	254T	256T	256T	256T	286T	324T
25	256T	284TS	281T	284T	324T	326T
30	284TS	286TS	286T	286T	326T	364T
40	286TS	324TS	324T	324T	364T	365T
50	324TS	326TS	326T	326T	365T	404T
60	326TS	364TS	364TS	364TS	404T	405T
75	364TS	365TS	365TS	365TS	405T	444T
100	365TS	405TS	404TS	405TS	444T	445T
125	404TS	444TS	405TS	444TS	445T	NA
150	405TS	445TS	444TS	455TS	NA	NA
200	444TS	NA	445TS	NA	NA	NA
250	445TS	NA	NA	NA	NA	NA

From MG 1-13.02a and 13.06a.
NA = Not Assigned.

Service Factor: This is a multiplier shown on the nameplate of general-purpose motors by the manufacturer. The nameplate horsepower rating multiplied by the service factor is an indication of the total load the motor can successfully carry when operated at standard temperatures, with unrestricted ventilation, and with rated voltage and frequency. The service factor is 1.15 for integral-horsepower motors with Class B insulation in sizes up to 200 hp (MG 1-12.47a).

When operating loads are above rated load and below maximum service-factor load, the motor will have an increased (but safe) temperature rise. Power factor, efficiency, and motor speed will also be different than for rated-load operation.

The service factor provides for those continuous loads which are only a few percent higher than a standardized horsepower rating. For example, a fan design may require 5 hp according to original estimates. The final production unit may require 5.25 hp. An open, general-purpose, 5-hp motor can still be used to handle the load (5-hp motor with a 1.15 service factor can be loaded up to 5.75 hp) without injurious overheating.

When motors are installed at altitudes higher than 3,300 ft above sea level (but less than 9,900 ft), the service factor may be used to compensate for the altitude. Loading should, of course, be nameplate horsepower rating or less.

Many installations do not have rated voltage (and occasionally not rated frequency) due to power-supply or plant-distribution problems. In these cases, the service factor may be used to compensate for actual installation conditions. Again, the loading should be nameplate horsepower rating or less.

Motor loads often fluctuate or follow a definite pattern of change during operation. For example, when a refrigeration compressor starts up, usually only a nominal amount of power is required early in the cycle. During the cycle, motor load increases predictably until the cutoff point is reached. The margin in the motor capacity represented by the service factor allows use of a motor with a size smaller than the horsepower required during the latter part of the cycle.

Polyphase Motors

Polyphase motors of three basic constructions are offered by most manufacturers. The most widely used type by far is the constant-speed squirrel-cage motor. Actually, with this motor, the speed varies slightly with load, but there is much less variation than there is with other sources of mechanical power, such as d-c motors and gasoline engines. However, multispeed squirrel-cage motors provide two or more discrete speeds.

A second construction has controllable speed over a fairly wide range when fully loaded. This is the wound-rotor motor. Another type of construction, the synchronous motor treated in Chapter 7, offers absolutely constant-speed operation.

Table 8—Locked-Rotor Torque of 60-Hz, Design A and B, Squirrel-Cage Motors*

Rating (hp)	3600	1800	1200	900
½	...	...	...	140
¾	...	...	175	135
1	...	275	170	135
1½	175	250	165	130
2	170	235	160	130
3	160	215	155	130
5	150	185	150	130
7½	140	175	150	125
10	135	165	150	125
15	130	160	140	125
20	130	150	135	125
25	130	150	135	125
30	130	150	135	125
40	125	140	135	125
50	120	140	135	125
60	120	140	135	125
75	105	140	135	125
100	105	125	125	125
125	100	110	125	120
150	100	110	120	120
200	100	100	120	120
250	70	80	100	100
300	70	80	100	...
350	70	80	100	...
400	70	80	...	...
450	70	80	...	...
500	70	80	...	...

From NEMA MG 1-12.37.
*In percent of full-load torque. Percentages are valid for 50-Hz motors, but speeds are 5/6 of 60-Hz speeds.

Squirrel-Cage Motors

Torque, horsepower, and speed requirements of a large number of applications can be satisfied with one of the four NEMA classifications, Table 7. Each design offers different torque, speed, and current characteristics to meet various operating requirements.

All designs can withstand full-voltage starting when connected directly across the power lines. They are mechanically strong enough to withstand the resultant magnetic stresses and locked-rotor torques at the instant the switch is closed. Most designs use schematic connections and standardized lead markings, Fig. 1.

Design A and B motors, in larger horsepower sizes and in lower synchronous speeds, develop lower percent torques than do the lower horsepower sizes and higher speeds, Table 8. Designs C and D are characterized by higher torques in all ratings. Pull-up torques, Table 9, are required to be higher for smaller-horsepower motors.

Standard breakdown torques are given in Table 10. There is less divergence from lowest to highest values in minimum required breakdown torque for Designs A, B, C, and D than for the locked-rotor torques. However, the speed at which maximum breakdown torque is developed is not the same for each of these designs.

Standard locked-rotor current limits are shown in Table 11. Larger motors are required to start on less current per horsepower.

Squirrel-cage motors are built in several classifications according to desired characteristics. A division may be made according to slip—less than 5% is defined as *low slip*, whereas *high slip* is 5% or more.

Low-slip motors are intended for loads that are relatively constant and run for long periods of time. These are Designs A, B, and C, Fig. 2. High-slip motors, NEMA Design D, are intended for fluctuating loads, or loads that are intermittent, Fig. 3.

Design A motors have higher breakdown torques and higher locked-rotor currents than Design B; but definite limits have not been standardized. Ap-

Table 9—Pull-Up Torque of 60-Hz, Design A and B, Squirrel-Cage Motors

Locked-Rotor Torque from Table 8	Pull-Up Torque
≤ 110%	≥ 90% locked-rotor torque
110 to 145%	≥ full-load torque
≥ 145%	≥ 70% locked-rotor torque

From NEMA MG 1-12.39.

Table 10—Breakdown Torque of 60-Hz, Design B, Squirrel-Cage Motors*

Rating (hp)	—Synchronous Speed (rpm)—			
	3600	1800	1200	900
½	...	...	...	225
¾	...	...	275	220
1	...	300	265	215
1½	250	280	250	210
2	240	270	240	210
3	230	250	230	205
5	215	225	215	205
7½	200	215	205	200
10-200	200	200	200	200
250	175	175	175	175
300-350	175	175	175	...
400-450-500	175	175	...	...

From NEMA MG 1-12.38.
*In percent of full-load torque. Percentages are valid for 50-Hz motors, but speeds are 5/6 of 60-Hz speeds.
Design A motor breakdown torques are in excess of those shown.

plications for these motors must be checked carefully because of the high locked-rotor current. Quite possibly, special controls may be required for starting. This motor is suitable for machines in which friction and inertia are small and starting and stopping are infrequent. Typical applications are hydraulic pumps and cutting tools such as saws and grinders. Customary range of Design A ratings is 1 to 500 hp at all speeds. Load requirements of typical applications fall within the following ranges: breakaway torque, 40 to 70% of full-load torque; accelerating torque, 20 to 50%; peak torque, 130 to 175%; load inertia, less than motor-rotor inertia. Continuous steady-load operation with infrequent (less than once an hour) starting or stopping is also typical.

Design B motors have normal starting torque and a starting current acceptable to most power systems. This design has relatively high breakdown torque and low slip. These motors are used on such applications as fans, machine tools, blowers, and centrifugal pumps. They will accelerate to full speed any load that the motor can start. Customary range is 1 to 500 hp at all speeds. Typical applications for this motor are those having a breakaway torque less than 50% of rated load; accelerating or pull-up torque requirements of less than 50%; occasional peak torque requirements not more than 125%; little or no torque pulsation; load inertia less than motor-rotor inertia; and continuous steady-load operation with infrequent starting or stopping. This motor and Designs A and C satisfy the drive requirements of about 75 to 85% of all motor-driven machines and equipment.

Table 11—Locked-Rotor Current Limits for Three-Phase, 230-v, 60-Hz, Design B, Squirrel-Cage Motors

Rating (hp)	Max Locked-Rotor Current (amps)	Code Letters
½	20	R
¾	25	P
1	30	N
1½	40	M
2	50	L
3	64	K
5	92	J
7½	127	H
10	162	H
15	232	G
20	290	G
25	365	G
30	435	G
40	580	G
50	725	G
60	870	G
75	1,085	G
100	1,450	G
125	1,815	G
150	2,170	G
200	2,900	G
250	3,650	G
300	4,400	G
350	5,100	G
400	5,800	G
450	6,500	G
500	7,250	G

From NEMA MG 1-12.34 and NEC 430-7 (b).
Locked-rotor current of motors designed for other than 230 v are inversely proportional to the voltage ratio.
Design A motor locked-rotor currents may be in excess of those shown. Limits shown apply to Design C motors from 3 to 200 hp and to Design D motors from ½ to 150 hp.
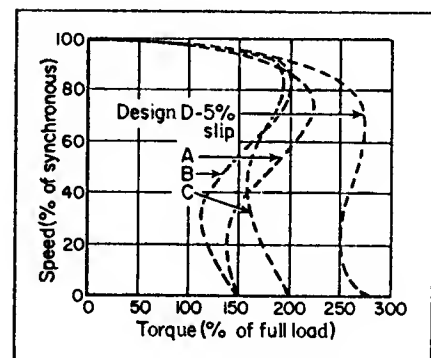

Fig. 2—Speed-torque curves for Design A, B, C, and D squirrel-cage motors rated at 20 to 30 hp.

Design C motors have a high starting torque and a normal breakdown torque. Typical applications for this design are machines in which breakaway loads are high at starting, but which normally run at rated load and are not subject to high overloads after running speed has been reached. Compressors that are

<u>EXHIBIT A-1</u>

not unloaded at start, plunger pumps, and many types of conveyors are typical examples of such machines. Ratings range from 7½ to 200 hp at 1,800 rpm; slightly lower horsepower at 1,200 and 900 rpm. Typical applications for this motor have load requirements in the following ranges: breakaway torque, 100 to 150%; accelerating torque, 75 to 125%; peak torque, 130 to 150%; load inertia, less than motor-rotor inertia; infrequent starting or stopping.

Design D motors develop high starting torque with moderate starting current. These motors also have over 5% slip, so that the speed can drop when fluctuating loads are encountered. Manufacturers have found it practical to subdivide this design into several groups which vary as to amount of slip (5 to 8%, 8 to 13%, etc.), or as to the shape of the speed-torque curve, Fig. 3. Typical applications for this motor are machines in which heavy loads are suddenly applied or removed at frequent intervals. Examples are hoists, cranes, punch presses, centrifuges, extractors, and machines with flywheels.
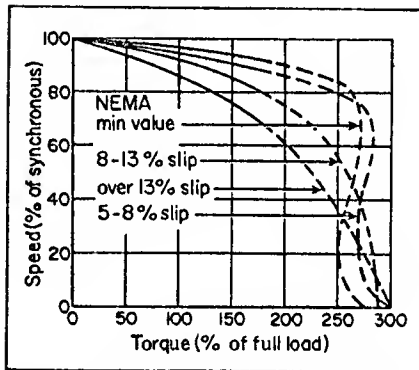
Fig. 3—Speed-torque relationship for Design D squirrel-cage motors.

Selection Factors

To properly select a motor, certain facts about both the motor and the application must be known. All NEMA limits in the curves and tables of this chapter are expressed in terms of the *application* or *load* limit. For example, if a 30-hp, 230-v motor is to have a maximum locked-rotor current of 435 amp and a minimum locked-rotor torque of 135 lb-ft (1,800-rpm motor), the motor user must provide for no less than 435 amp and may demand 135 lb-ft

torque for starting—but no more.

Because this much performance is specified for the user, it is customary for the motor manufacturer to exceed the minimum values (such as torques) and fail to take advantage of the maximum values (such as slip or currents or temperature rise) in the furnished motor. By this arrangement, the user is assured of a definite performance and the manufacturer can arrange his designs, manufacture, and test limits to satisfy the user.

Required characteristics and functions of the load or driven machine which must be known for correct motor selection and application are:

Horsepower: The required horsepower should be a realistic value. Often, horsepower requirements increase as the driven machine becomes worn. Expected wear in parts or variation because of a change in lubrication efficiency between maintenance periods should be considered and averaged for the life-expectancy of the machine when determining horsepower.

Speed: In general, higher-speed motors are smaller and less expensive. For this reason, many machines have arrangements of belting, gearing, or other speed-changing drives so that the drive motor may be chosen from the less expensive and more readily available 3,600, 1,800, or 1,200-rpm speed classes.

Load Variations: Many machines have continuous loads over long periods of use, such as fans or blowers. Other machines have wide variations in running load, such as deep-draw presses which have a flywheel. Often, the flywheel supplies the energy to do the work and the motor merely restores lost energy to the flywheel. Whether the load is steady, varies from piece to piece, follows a repetitive cycle of variation, or has pulsating torques or shocks must be considered.

Peak Loads: The largest expected momentary or short-time overload for the driven machine should be determined. For low-slip motors, this overload is usually expressed in percent of normal horsepower. For machines which are driven by high-slip motors, this ratio should be expressed in percent of torque rather than horsepower.

Machine Inertia: Not only should the inertia of the driven machine and the proposed motor be known, it is important to also know if the inertia changes during the work cycle. For example, in some extractors handling

batches of material, the purpose of the machine is to separate and remove certain unwanted material during the work cycle. This reduces the load inertia.

Breakaway Torque: This is the sum of the static friction torque of the load, plus the kinetic friction torque (or the torque required at very low speeds) of the load. The static-friction torque of the load is a result of all the machine and motor friction losses, such as losses caused by any pistons and valves, and pressures which may remain in compressor cylinders. Kinetic friction is that due to the inertia of all the parts which must be started from rest and given momentum. Machines which are subject to accumulations of dirt and ice—or have poor maintenance or lack of lubrication—have higher breakaway torques than clean, well-lubricated machines. This factor should not be underestimated.

Frequency of Starting or Stopping: Since starting energy is so much higher than steady-state operation, this is an important consideration for overheating. Many fans and blowers are allowed to run continuously for hours or days, whereas automatically controlled compressors and pumps start a number of times per hour. Frequently, machine-tool motors have many starts or stops per minute.

Starting-Current Limitations: In many motor applications, the amount of available starting current is limited. Usually, limits are set by the utility supplying power, but inadequate wiring and fusing may also limit starting current.
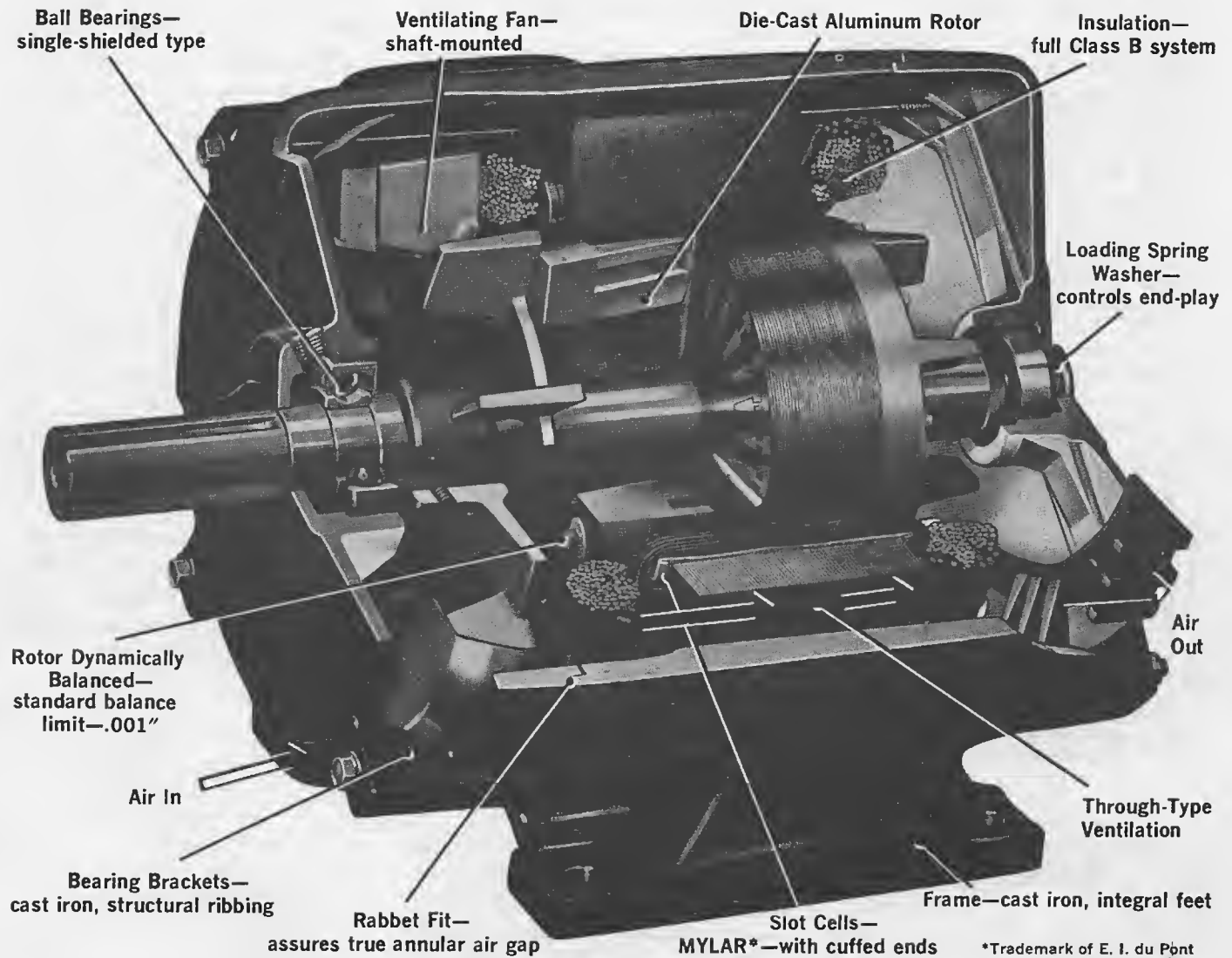
Ball Bearings—
single-shielded type

Ventilating Fan—
shaft-mounted

Die-Cast Aluminum Rotor

Insulation—
full Class B system

Loading Spring
Washer—
controls end-play

Rotor Dynamically
Balanced—
standard balance
limit—.001"

Air In

Air
Out

Through-Type
Ventilation

Bearing Brackets—
cast iron, structural ribbing

Rabbet Fit—
assures true annular air gap

Slot Cells—
MYLAR*—with cuffed ends

Frame—cast iron, integral feet

*Trademark of E. I. du Pont

EXHIBIT A-2

Typical Squirrel Cage Induction Motor Construction

TIME-TO-START (B)

George Boone reviewed his calculations from a year earlier (Exhibit B-1). He had simply added the WR^2 of the load to the WR^2 of the motor itself. In this case the motor was started at no load. All the motor torque would be used to accelerate the machinery. He had simply assumed that the torque was constant throughout the start up time, first equal to running torque, then equal to maximum torque. He established that the unit would be up to full speed between 3-1/2 to 7-1/2 seconds.

George now turned his attention to the problem at hand, the Seldon fan motor. The calculations could be carried out in a similar manner except that the fan would be drawing power while it was accelerating. He did not know speed-torque characteristics for the fan but he knew it was not linear. It was going to be some power function of the speed.

Because an answer was needed in a hurry he did not want to waste time on speculation as to the exact speed-torque characteristics of the fan. George decided to assume that the torque required was proportional to speed. This should give him a final result on the conservative side.

150 H.P. 900 R.P.M. 550/3/60 S.C.I. Motor.

Design #1875   844 Frame.
WR^2 of attached parts = 1950 # ft^2
Rotor Dia = 25.61 in.
Starting Torque = 125%
Max. Torque = 200%

Determine time for motor with attached parts
to get up to speed.
  With no load applied time will be between
time with running torque and time with
maximum torque.

Running Torque $T = \dfrac{150 \times 33000}{2\pi \times 900} = 876$ # ft.

Max Torque = $876 \times 2 = 1752$ # ft.

k^2 for 25.61 in Rotor = $\dfrac{12.8^2}{4 \times 144} = .285$ ft^2

Assume Rotor Wt = 1000 #

  WR^2 of Rotor = $1000 \times .285 = \underline{285}$ # ft^2

Total WR^2 acting = $1950 + 285 = 2235$ # ft^2

I (Moment of Inertia) $= \dfrac{WR^2}{g} = \dfrac{2235}{32.2} = \underline{69.4}$ # ft^2

$T = I\alpha$

$\alpha = \dfrac{T}{I}$

EXHIBIT B-1

Facsimile

For Running torque

$$\alpha = \frac{876}{69.4} = 12.7 \text{ rad/sec}^2$$

For Max torque

$$\alpha = \frac{1752}{69.4} = 25.4 \text{ rad/sec}^2$$

ω_t = final angular velocity
ω_0 = Initial angular velocity

$$\omega_t = 900 \text{ rpm} = \frac{900 \times 2\pi}{60} = 94.3 \text{ rad/sec}.$$

$$\omega_0 = 0$$

$$\omega_t = \alpha t + \omega_0$$

$$t = \frac{\omega_t}{\alpha}$$

For Running torque

$$t = \frac{94.3}{12.7} = \underline{7.43 \text{ sec.}}$$

For Max torque

$$t = \frac{94.3}{25.4} = \underline{3.71 \text{ sec.}}$$

∴ $7\frac{1}{2}$ to $3\frac{1}{2}$ sec is required to bring unit up to speed.

EXHIBIT B-1

TIME-TO-START (C)

To determine the starting time for the Seldon fan motor George Boone approximated his fan speed torque curve by a straight line, zero start and 85 h.p. at 3520 rpm. He plotted this relationship and the approximate speed torque curve for the motor on squared paper (Exhibit C-1). He estimated the motor speed torque curve using locked rotor torque, breakdown torque, full load torque and zero torque at synchronous speed.

Visual examination of these two speed torque curves fortunately showed that the torque available for acceleration was approximately constant. Therefore, by determining the area between the curves he arrived at an average acceleration torque. He used this average torque and found the starting time was approximately 10 seconds.

Consultation with the motor designers told him that there was no probability of motor burn out during this time. George therefore called Kirkland and passed the information to him. Kirkland was able to confirm the order with Seldon the same afternoon on which the inquiry was made.

George was aware that there were some severe approximations and said to himself that some day when he had free time he would look for a more accurate method. Of course other things became more pressing and he never did.

Facsimile

To determine the starting time for a 100 H.P. 3600 R.P.M. ~~6ye~~ 505 machine driving a fan whose $WR^2 = 90$ lbs ft².

WR^2 of Motor.

    WT Rotor = 317 #

    R^2 of Rotor = 18 in²

$W R^2$ of Rotor = 38.4 # ft²

    WT Shaft = 59.4 #

    R^2 of Shaft = 1.1 in²

Use 40 # ft² for WR^2 of Motor.

Total WR^2 of Rotating Part = 90 + 40 = 130 # ft²

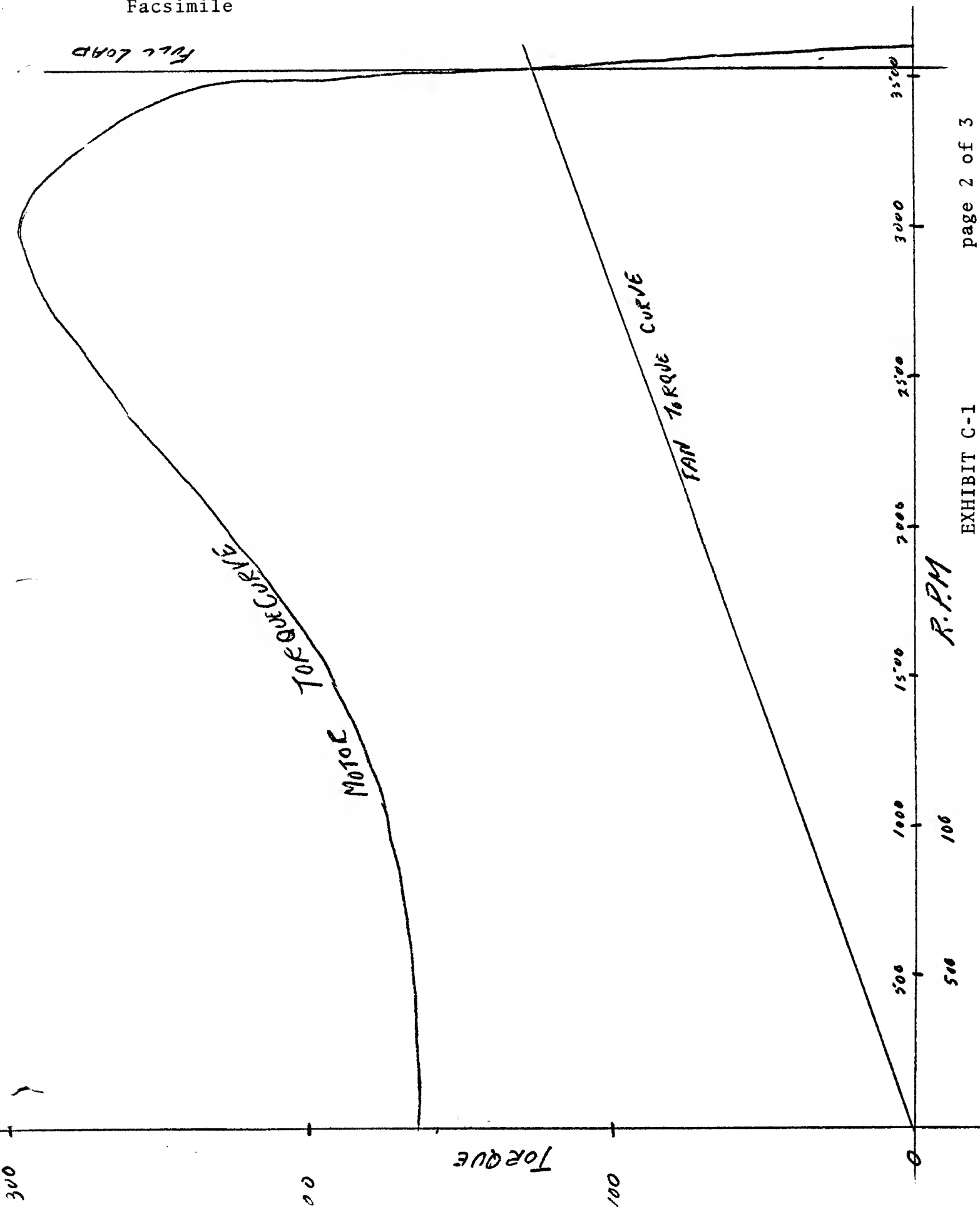Motor Full load Torque = $\dfrac{100 \times 33000}{2\pi \times 3520}$ = 149 ft #

85 H P Fan at 3520

    Fan Torque requirement = $\dfrac{85}{100} \times 149$ = 127 ft #

Max motor Torque 200% = 294 ft #

Motor Starting Torque 110% = 164 ft #

EXHIBIT C-1

EXHIBIT C-1

Facsimile

Area between Motor Torque Curve and Fan
Torque Curve = 34.7 in^2

Constant = 16000 ft$^\#$ rpm / in^2

Average Accelerating Torque = $\dfrac{34.7 \times 16000}{3520}$ = 158 ft$^\#$

$\propto = \dfrac{158 \times 32.2}{130}$ = 38.1 rad/sec^2

$W_2 = \dfrac{3520 \times 2\pi}{60}$ = 368 rad/sec.

$\propto t = W_1 - W_2$

$t = \dfrac{368}{38.1}$ = 9.66 sec.

Therefore time for motor to bring fan up
to speed if voltage is maintained will be
approx. 10 sec.

EXHIBIT C-1

page 3 of 3

INSTRUCTOR'S NOTE

Time-To-Start

This case is intended to be used as a case problem in the context of a real engineering situation. It shows the application of the elementary mechanics of rotating bodies. It should also give some insight into the characteristics of induction motors.

The case has been broken into three parts. Each may be assigned in such a manner as to lead the student through the various stages of analysis.

1. An upper and lower bound solution
2. Graphical approximation solution
3. A more exact pricewise integration not covered in the case.

The case can be used to show that the degree of sophistication of the model can be tailored to the specific requirements.

The instructor may assign or demonstrate to the class the more exact solution by a more detailed analysis of the problem. This type of problem is discussed in "Engineering Analysis" by Ver Planck and Teare, published by John Wiley and Sons, Chapter 5. The technique illustrated can be applied to the problem in the case.

The results of the more exact analysis can then be used to evaluate the accuracy of George Boone's various approximations.

If students are required to work all three methods they can ge given an opportunity to measure the time involved in mastering and solving the problem at the different levels of accuracy.

5-2   A Graphical Solution: Starting Time of a Fan

To begin a study of the steps in solving a differential equation let us consider a problem of predicting from design information how long it will take a fan driven by a motor to come up to speed after the motor is energized. The fan is of the centrifugal type and is to be driven by a three-phase wound-rotor induction motor rated at 10 horsepower. Starting of
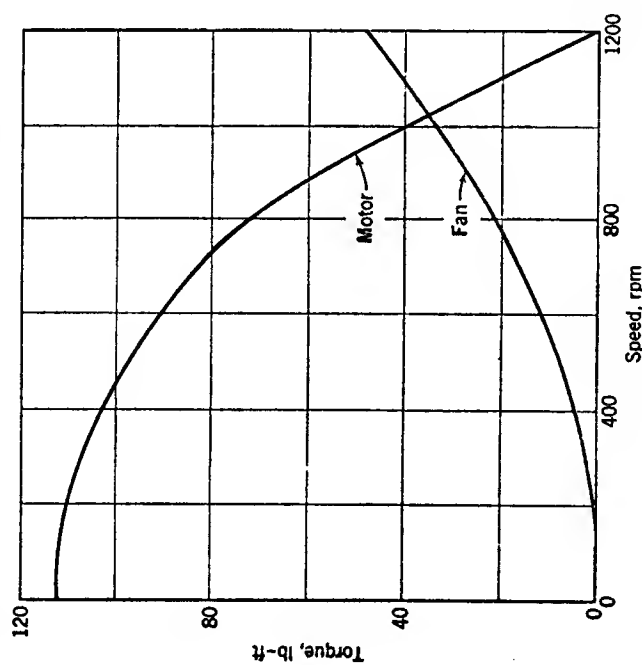
the motor is to be at full voltage and with resistance connected in the rotor circuit. We wish to find how the speed varies with time during the starting period, the rotor resistance remaining unchanged.

The predicted torque-speed curve for the fan, that is, the relation between the speed and the torque required to drive the fan steadily at that speed with the anticipated discharge conditions, is shown in figure 5-1. Also shown in the figure is the torque-speed curve for the motor. This curve shows the torque which the motor delivers at its coupling as a function of speed. The steady operating speed of the fan and the motor will be at the point of intersection of the motor curve and the fan curve because

at this speed the torque supplied by the motor is just equal to that required to keep the fan running. Moreover, operation at this speed will be stable, for at any lower speed there will be an excess of driving torque which will cause the system to accelerate, and conversely at any higher speed there will be a deficiency of driving torque and the system will decelerate, in both cases approaching the intersection point. Thus examination of the curves in figure 5-1 shows that the steady running speed of the fan under these conditions will be 1020 rpm.

In planning how to find the variation of speed with time we begin by considering what happens as the motor is started. In the first place, there
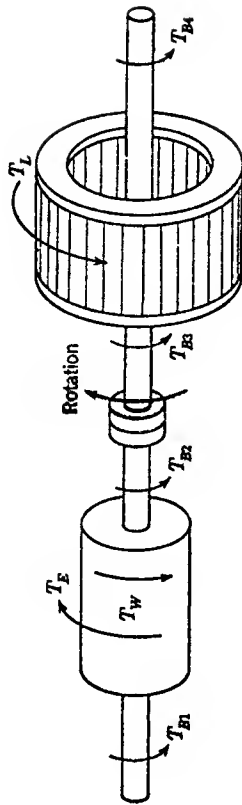

Fig. 5-2.. Free-body diagram of the rotor showing the torques.

may be an electrical delay while the current rises from zero to some value before the motor begins to turn. Then, or possibly simultaneously, there is the interval in which the rotating parts are accelerating under the influence of the motor and load torques. Experience leads us to expect the electrical transient in this problem to be much shorter in duration than the mechanical transient, and for simplicity we assume the electrical transient to be negligible. We assume that at any speed the motor produces the torque given by the curve in figure 5-1 even though this curve is for the motor running at constant speeds with steady-state current and in the present situation the speed is changing with time.

Having reduced the problem to a mechanical one, we draw a free-body diagram, figure 5-2, which shows the rotors, shafts, bearings, and coupling, together with the torques that act upon the rotating system. These are the electromagnetic torque of the motor, the load torque of the fan, the bearing torques, and the wind-friction torque exclusive of the fan. According to the law for the angular acceleration of a rigid body, that is, $\Sigma T = I\alpha$, the resultant of the torques of motor, load, bearings, and wind friction is equal to the product of the moment of inertia of the whole system by its angular acceleration. This is the verbal statement which gives the application of the fundamental principle to our particular problem. We expect that it may be possible to manipulate the relation to yield a speed-time curve.

Fig. 5-1.   Torque-speed curves for fan and motor.

In the execution stage, the verbal statement is first expressed in mathematical language.

Let

T_E = electromagnetic torque acting on the rotor, positive when it has the direction of rotation (lb ft).

T_L = load torque of the fan, positive when it has the direction opposite to that of rotation (lb ft).

$T_{B1}, T_{B2}, T_{B3}, T_{B4}$ = bearing friction torques, with the same positive directions and units as T_L.

T_W = wind friction torque exclusive of the fan, with same positive direction and units as T_L.

I = moment of inertia of whole system, (slug ft²).

ω = angular velocity, positive in the actual direction of rotation, (radians sec⁻¹).

t = time with zero at the instant at which the motor is energized (sec).

Then, $\qquad T_E - T_{B1} - T_{B2} - T_W - T_L - T_{B3} - T_{B4} = I\dfrac{d\omega}{dt}$ (5-1)

Some simplification may be made by recognizing that $T_E - T_{B1} - T_{B2} - T_W$ is the torque available at the shaft of the motor when there is no acceleration. This will be designated by T_M; that is, it is the torque that the motor will deliver from the end of the shaft at constant speed and is given by figure 5-1. For this statement to be strictly true, T_W, which was defined as all the wind friction exclusive of the fan, must be the same as the wind friction within the motor. In other words, we neglect any wind friction on the coupling and connecting shaft. Likewise the torque required at the coupling to turn the fan at steady speed is $T_L + T_{B3} + T_{B4}$, designated by T_F. Then

$$T_M - T_F = I\frac{d\omega}{dt}$$ (5-2)

Although we might have been tempted to write this equation sooner, it is important to note, that proceeding as we did, we know precisely how and to what extent the frictional torques are accounted for; otherwise there would have been room for doubt; likewise we are sure which moment of inertia is involved.

Equation 5-2 is a differential equation; we come to the problem of how to solve it. First let us summarize what we know about the terms it contains. The motor shaft torque T_M is a function of ω (and for an induction motor in the steady state electrically, as we are assuming here, a function of ω only); it is in fact the function represented by the motor curve in figure 5-1. Therefore we may regard this function of angular velocity, $T_M(\omega)$, as known. Likewise T_F, the torque to drive the fan, is a function of angular velocity, $T_F(\omega)$, and is known from the fan torque-speed curve of figure 5-1. Thus, the entire left-hand member of equation 5-2 is a known function of angular velocity ω; that this function happens to be given by curves instead of by a mathematical formula is not important. The right member of 5-2 contains I, the moment of inertia of the whole rotating system, and this we have estimated from drawings by methods which we will not discuss here to have the value 1.26 slug ft². Also in the right member are ω, the dependent variable which is sought, and t, the independent variable.

The equation can be regarded as giving the time rate of change of ω for any value of ω, thus using functional notation

$$\frac{d\omega}{dt} = \frac{T_M(\omega) - T_F(\omega)}{I}$$ (5-3)

Our problem is to find a function of t, $\omega(t)$, such that its slope $\dfrac{d\omega}{dt}$ depends on ω in accordance with the right side of 5-3. The right side of 5-3 is a function of ω which could be plotted against ω by using the data of figure 5-1. Then, if we know the value of ω at some value of t, and we do know the initial condition that $\omega = 0$ at $t = 0$, we can calculate the corresponding value of $\dfrac{d\omega}{dt}$. This suggests that over a short enough interval of time, Δt, we can find the change of ω with reasonable accuracy by multiplying $d\omega/dt$ by Δt, thus:

$$\Delta\omega = \omega_1 - \omega_0 \doteq \frac{d\omega}{dt}\,\Delta t = \frac{T_M(\omega_0) - T_F(\omega_0)}{I}\,\Delta t$$ (5-4)

in which ω_0 is the value of ω at the beginning of the interval, ω_1 at the end. Thus, by starting with $\omega_0 = 0$, we could find ω_1 at the end of the small interval Δt. For the next interval Δt, we could proceed similarly except that ω_0 would be replaced by the value $\omega = \omega_1$ just found and a new value. $\omega = \omega_2$, would be found for the end of the second interval. By repeated calculation, we could construct a broken line curve giving ω as a function of t, as sketched in figure 5-3.

How can we tell how small to make the interval Δt? One way would be to make an arbitrary choice, say 0.1 sec, calculate a portion of the curve, then try a smaller value, say 0.05 sec, repeat the calculation, and compare the results of the two calculations. If the results differ significantly, the first interval was too large, and perhaps the second is, also; in this case, we

where, since ω is to correspond to t, the upper limit on the right side is ω and the lower limit is zero provided ω = 0 at $t = 0$. Thus the initial condition enters this solution too! If we can integrate the right side, we have a solution. It should certainly be possible to integrate it graphically or numerically, for $T_M(\omega)$ and $T_F(\omega)$ are known, hence we should be able to draw a curve of $\dfrac{1}{T_M(\omega) - T_F(\omega)}$ as a function of ω, as sketched in figure 5-4. According to equation 5-6, the time required for the fan to come up
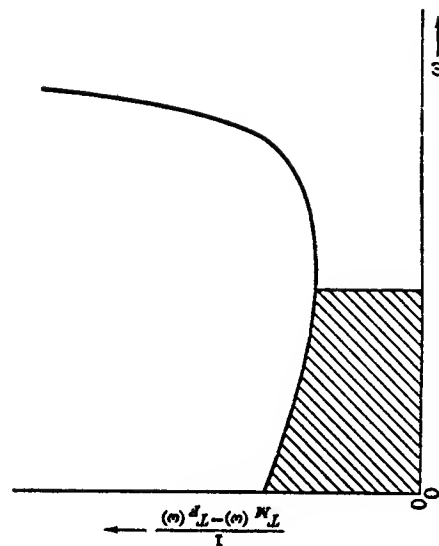
Fig. 5-4. Rough sketch of the function to be integrated in equation 5-6.

to any speed ω is the area under the curve up to the vertical line through ω, multiplied by I. Thus, we have a plan for solving 5-2 which appears to be less tedious and perhaps more accurate than the step-by-step procedure worked out first.

To carry out the integration we replace the curve which was sketched in figure 5-4 by one plotted accurately on rectangular coordinate paper, figure 5-5, and which for simplicity is in terms of speed N in rpm instead of ω. Then we replace $d\omega$ in 5-6 by $\dfrac{2\pi}{60} dN$ and obtain

$$t = \frac{2\pi}{60} I \int_0^N \frac{dN}{T_M(N) - T_F(N)} \tag{5-7}$$

where t is still the time in seconds.

The area can now be found by counting squares or with the aid of a planimeter. The authors' experiences lead them to feel that, unless a large number of areas are to be determined, it does not pay to find a

should try a still smaller interval. If the results are in good agreement, the interval is small enough, possibly unnecessarily small (to our regret, since the calculation is tedious), but at least it is on the safe side.

We now have one way of solving the differential equation. Our procedure illustrates very clearly the kind of relation that a differential equation represents, and that to get a particular solution we have to know ω at some value of t, called an initial condition. Observe that the same differential equation would hold if the problem were somewhat different, for
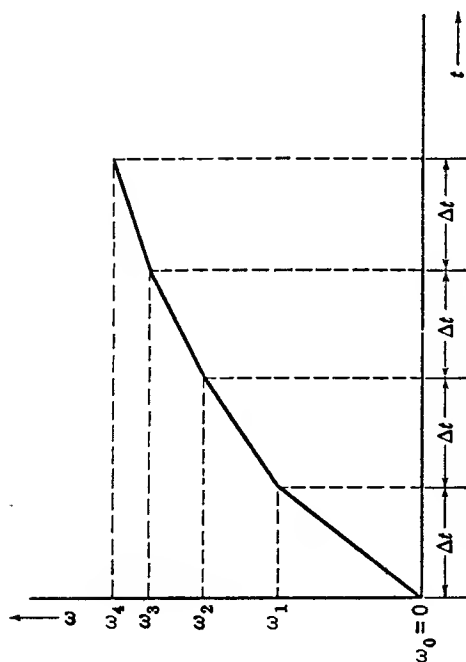
Fig. 5-3. Illustrating a step-by-step process for solving equation 5-3.

example, to find the speed-time curve following a sudden change in rotor resistance to the value corresponding to figure 5-1 from some other value with the motor already running. Although the differential equation would be the same, the initial condition would be different and a different curve would be found for the solution.

Although we have a straightforward method for solving the differential equation, we are not pleased with it because of the apparent tediousness of ascertaining that the intervals Δt are small enough. So we try another approach to see whether anything can be gained by separating the variables ω and t, thus:

$$dt = I \frac{d\omega}{T_M(\omega) - T_F(\omega)} \tag{5-5}$$

We observe that t comes from dt by integration and try

$$t = \int_0^t dt = I \int_0^\omega \frac{d\omega}{T_M(\omega) - T_F(\omega)} \tag{5-6}$$

Speed N (rpm)	Squares for last increment of N	Total squares up to N	t (sec) to reach N = (squares) × 0.5 × $\frac{2\pi}{60}$ × 1.26
0	0	0	0
100	1.8	1.8	0.12
200	1.8	3.6	0.24
300	1.9	5.5	0.36
400	2.0	7.5	0.50
500	2.15	9.65	0.64
600	2.35	12.0	0.79
700	2.8	14.8	0.98
800	3.5	18.3	1.21
900	5.2	23.5	1.55
950	4.3	27.8	1.83

Before using figure 5–6, the result of our analysis, we make some overall checks. (The arithmetic, of course, was checked carefully as it was performed.) In the first place, based on experience with machines of this general size and character, 2 sec to reach 90 per cent of full speed seems to be of the correct order of magnitude. Had our result been a few milliseconds or several minutes, we would have had good grounds for suspecting a gross error of some sort.

Fig. 5-6. Speed-time curve calculated by taking areas under the curve of figure 5-5.

planimeter and learn or relearn the technique of using it; the same may be said of formula methods such as Simpson's rule. Counting squares is fully as accurate and less costly in time than the more sophisticated methods.

Counting squares, we find, for example, the time to reach 100 rpm. The area under the curve up to 100 rpm we estimate to be 1.8 squares. The
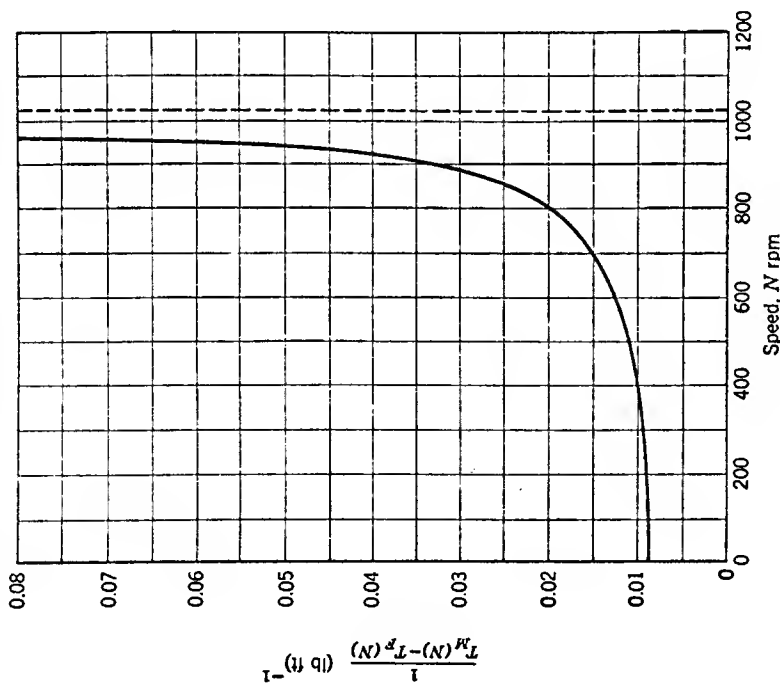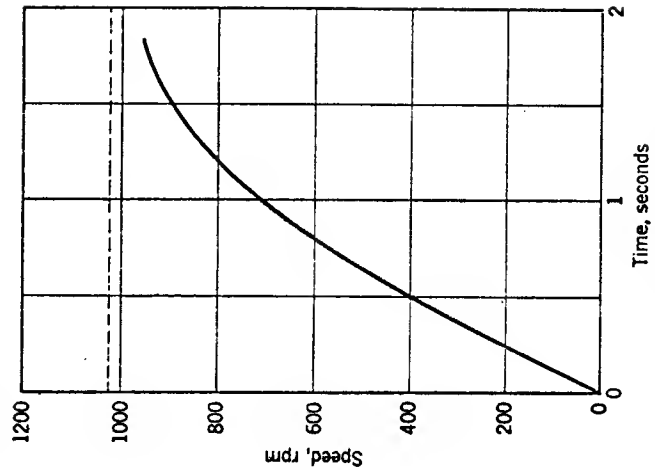
Fig. 5-5. Plot of the integrand in equation 5-7.

value of a square is dependent on the scales to which the curve is plotted and in this case is 100 × 0.005 = 0.5. The integral up to 100 rpm is then 1.8 × 0.5 = 0.9, and the time to reach 100 rpm is the product of this number by the value of I (=1.26) and $2\pi/60$ or $0.9 \times 1.26 \times 1.26 \times 2\pi/60 = 0.12$ seconds. Next, we count the squares between 100 and 200 rpm, add them to those already counted, and get the time to reach 200 rpm as before. Proceeding in this way, we construct the accompanying table and draw the speed-time curve, figure 5–6, and so attain the goal we set for ourselves.

Now we make a more specific check. Examining the original data in figure 5–1, we observe that the resultant torque available for acceleration up to 500 rpm is never far from 100 lb ft. If it were exactly constant at this value, T_a, the time to reach 500 rpm would be

$$t = \frac{I\omega}{T_a} = \frac{1.26 \times 2\pi \times 500}{100 \times 60} = 0.66 \text{ sec} \qquad (5\text{–}8)$$

and this agrees closely with the value calculated in our table. Further examination of figure 5–1 shows that the torque available for acceleration above 500 rpm is always much less than 100 lb ft. and therefore the time to accelerate the rest of the way to full speed should be much more than 0.66 sec, which also agrees with our analysis.

The preceding argument suggests that the whole analysis could have been made more easily by simply averaging the torque available for acceleration with respect to speed and then making a calculation like 5–8. It is true that in problems like this there is always some way of taking an average that will give a correct answer, but the correct average is often not that found by taking the arithmetic mean. Here, for instance, it is clear that although the simple average of torque with respect to speed is good enough for a rough check it is not really accurate; the correct kind of average is found by averaging the reciprocal of net torque and then taking the reciprocal of the result. Thus in general the use of averages is a questionable procedure until the problem has been solved without them in order to find out what kind of an average to use.

Let us see now what we can learn from this illustration that may be generally useful in solving differential equations. In the first place, basing our reasoning on physical pictures of the relationship given by the differential equation, we were able to plan two entirely different methods for solution, both perfectly straightforward. For the first method we thought of the desired result $\omega(t)$ as a graph of ω versus t and of the differential equation as defining the slope of this curve. Then the plan was to step our way along the curve in a series of short straight-line segments, each having the slope given by the differential equation for the value of ω at that point. To start this process we had to know the value of ω at some one value of t; and we did know the initial value. The second method we planned, and this is the one we adopted, was possible in this case because the variables in the differential equation were separable so that the time t could be expressed as an integral with respect to ω of a function of ω. Here too, we reasoned in terms of physical pictures, thinking of the integral as an area under a curve; and in this case, since the data were known in curve form, we performed the integration by actually finding an area under a curve plotted to scale. In this second method, as in the first, we found it necessary to know the value of ω at one value of t in order to get started.

Thus in general we can see that the differential equation by itself did not suffice to determine the solution to our particular problem; something else was needed, the fact that the fan starts from rest, that is, $\omega = 0$ at $t = 0$. In other words the differential equation can be satisfied by any one of a whole family of solutions—curves of ω versus t—each starting at a different initial value of ω. To find the particular solution we want, we must determine the appropriate initial conditions from physical relationships which are not contained in the differential equation. Not only is this conclusion valid in this instance, it is also true in general of the solutions of all differential equations. We can see also that for this first-order differential equation, one in which the highest derivative is the first as in this illustration, only one condition was needed to determine a particular solution, and this is true of all first-order differential equations.